Effect of Wing Configuration on Lift at Supersonic Speeds

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Abstract—This paper is to discuss the effect of each wing parameter on the lift-curve slope, based on the results of a parametric study conducted using both analytical and semi-empirical approaches. This paper also serves as a tool to evaluate these two approaches in order to gauge their respective feasibility and to use the most appropriate method to generate a design table that correlates the investigated parameter.

Keywords—supersonic, wing, configuration, lift coefficient, Mach number

1. Introduction

Supersonic wings are generally used in flying bodies as a means of control and stability as well as to maintain flight much like a subsonic wings; however, there are multiple configurations to design a supersonic wings to which must be taken into account during the design process as each configuration has its own strength and weaknesses and are only suited for a particular scenario based on a set of Tactical-Technical Requirement (TTR).

On the very fundamental level, wings are designed and developed solely to produce lift whether it is in the supersonic regime (Mach 1.2~4) or subsonic region. The aim of this paper is to discuss the effect of several wing parameters and their configurations to develop a design table that will instrumental for preliminary wing design.

2. Methodology

The variables that are investigated in this study are the wing geometrical characteristics and flight conditions. These variables are:

a. Airfoil shape
b. Relative thickness, \( t \)
c. Aspect ratio, \( A \)
d. Leading edge sweep angle, \( \Lambda_{LE} \)
e. Taper ratio, \( \lambda \)
f. Mach number, \( M \)
g. Lift Coefficient, \( C_L \)

In order to simplify the analysis, the angle of attack is assumed to be zero or small (below the stall angle) and the lift coefficient is expressed in terms of \( C_{L_{\alpha}} \) or lift force curve slope per radians. The objective is to correlate the effect of these parameters on the generated lift. It is postulated that some of these parameters have a greater effect on the generated lift compared to others. Hence for the purpose of this study, four types of wing plan forms were studied as shown in Fig.1

![Fig. 1 The investigated wing planforms (a) rectangular unswept (b) swept (c) delta (d) trapezoidal](image)

The effect of airfoil shapes were also taken into consideration in which two shapes, biconvex and double wedge were considered as shown in Fig. 2

![Fig. 2 The investigated airfoils (a) biconvex (b) double wedge](image)

There are 2 main approaches discussed within this paper, which are:

a. Analytical/ Theoretical Approach
b. Semi-Empirical Approach

2.1 Analytical Approach

The first method is to use existing theoretical formulations based on the Linearized Supersonic Theory [1] and the Busemann’s Second Order Approximation [1]. Based on these two theories, two models were obtained:

\[ C_{L_{\alpha}} = \frac{4}{\sqrt{M^2 - 1}} \]  

Equation (1) is the simplified equation obtained from the linearized theory. On the fundamental level, this theory only considers \( C_{L_{\alpha}} \) as a function of Mach number, and it does this by assuming the wing to be of infinite span and of infinitesimally small thickness.

The Busemann’s approximation expands Eqn. (1) by stating that the wing is of finite span and of finite thickness as expressed by Eqs (2) to (3)

\[ C_{L_{\alpha}} = \frac{4}{\sqrt{M^2 - 1}} \left[ 1 - \frac{1}{2(A(\sqrt{M^2 - 1}) - (1-C_A'))} \right] \]  

\[ C_L = \frac{\gamma M^4 + (M^2 - 2)^2}{2(M^2 - 1)^3} \]  

Equations (2) and (3) also takes into account the aspect ratio as well as the airfoil shape which is denoted by \( A' \) which are either two-thirds or half of the relative thickness corresponding to biconvex and double wedge respectively.

Analytical equations (1) and (2) were tested based on a series of cases based on Table 1 and benchmarked against wind tunnel results [2] for the purpose of validation.

<table>
<thead>
<tr>
<th>Table 1. Wing characteristics and flight conditions</th>
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<tbody>
<tr>
<td>Design Parameter</td>
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<tr>
<td>Aspect ratio, ( A )</td>
</tr>
<tr>
<td>Taper ratio, ( \lambda )</td>
</tr>
<tr>
<td>Thickness ratio, ( t )</td>
</tr>
<tr>
<td>Mach Number, ( M )</td>
</tr>
<tr>
<td>Leading Edge Sweep angle, ( \Lambda_{LE} )</td>
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</table>
2.2 Semi Empirical Approach

The semi-empirical approach is the utilization of existing graphs [3] to generate a matrix array which is to be incorporated into a MATLAB sub-routine (MATLAB program). The sub-routine utilizes the interpolation method to determine the $C_{L\alpha}$ based on the same set of parameters as outlined in Table 1. The semi empirical approach is to serve as an alternate means to determine the values of $C_{L\alpha}$ in which will later be used to compare with results obtained from analytical formulations. This is to validate the results obtained throughout the study.

3. Results and Discussions

Initially it is suspected that the aspect ratio, $A$, taper ratio, $\lambda$, and leading edge sweep angle, $A_{LE}$, would have a more prominent effect on $C_{L\alpha}$ compared to the other parameters [5]. However based on the results shown in Table 2, it was found that one of these parameters which is the taper ratio has no significant impact on the lift.

![Fig. 3 Comparison of airfoil shape performance](image)

<table>
<thead>
<tr>
<th>Table 2. Aerodynamic Design table</th>
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<tbody>
<tr>
<td>Design Parameter</td>
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</tr>
<tr>
<td>Aspect Ratio</td>
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<tr>
<td>Taper Ratio</td>
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<tr>
<td>Relative Thickness</td>
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<tr>
<td>Leading Edge Sweep Angle</td>
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<td>Airfoil</td>
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This discovery was an anomaly considering that taper ratio plays an important role in determining the shape of the wing, especially the trapezoidal planform, however it was later found that the taper ratio and leading edge sweep angle are highly coupled. This implies that the taper ratio cannot be changed without changing the leading edge sweep angle at the same time which means the effect of taper ratio is somewhat tied to the effect of the leading edge sweep angle. However the anomaly here is that the leading edge sweep angle can be changed freely without altering the taper ratio. This indicates that the taper ratio is dependent on the sweep angle whereas the sweep angle itself is independent of the taper ratio.

The leading edge sweep angle itself also has a profound effect on the lift, which is that at low Mach numbers, an increase in swept angle would degrade the performance of the wing; however at higher Mach numbers, the increase in swept angle has a positive effect. The reason for such behavior still requires further investigation.

The relative thickness and airfoil shape were also found to have no significant effect on the lift although it is speculated that these two parameters influences the drag instead [3,4]; however this effect was not studied as it was beyond the scope of the study. Further investigation was conducted to determine the effect of the airfoil shape on $C_{L\alpha}$. As seen in Fig.3, there are no clear indication that either of the airfoil shapes considered has any advantage over each other, however at low Mach numbers the biconvex airfoil delivers a slightly better performance compared to the double wedge profile. Although in spite of this performance gain, the double wedge profile is more commonly used compared to the biconvex as it is easier to manufacture a double wedge airfoil compared to a biconvex airfoil[5].

4. Conclusion and Recommendations

Based on the outlined parameters in Section 2, a comprehensive study has been conducted based on two main approaches and the results have been tabulated into a design table shown in Table 2. It is evident that only the aspect ratio, $A$ and leading edge sweep angle, $A_{LE}$ have an impact on lift. However the credibility of the results is still questionable and is only suitable for preliminary design. Further study using other methods such as numerical approach and experimental approach are still required to further validate the effect of each parameter as well to understand the coupled effect of both taper ratio, $\lambda$ and leading edge sweep angle, $A_{LE}$.

References